

# Electroweak Instantons, Axions, and the Cosmological Constant

Larry McLerran<sup>(1,2)</sup>, Robert D. Pisarski<sup>(1,2)</sup>, and Vladimir Skokov<sup>(1)</sup>

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1. Nuclear Theory, Department of Physics, Brookhaven National Laboratory, Upton, NY 11973, USA
2. RIKEN BNL Research Center, Brookhaven National Laboratory, Upton, NY 11973, USA

## Abstract

If there is explicit violation of baryon plus lepton number at some energy scale, then the electroweak theory depends upon a  $\theta$ -angle. Due to a singular integration over small scale size instantons, this  $\theta$ -dependence is sensitive to very high momentum scales. Assuming that there is no new physics between the electroweak and Planck scales, for an electroweak axion the energy difference between the vacuum at  $\theta \neq 0$ , and that at  $\theta = 0$ , is of the correct order of magnitude to be the dark energy observed in the present epoch.

## 1 Introduction

In a gauge theory, a  $\theta$ -angle appears by adding a term

$$\theta \frac{\alpha}{8\pi} \int d^4x \operatorname{tr} \left( F_{\mu\nu} \tilde{F}^{\mu\nu} \right) \quad (1)$$

to the action, where  $F_{\mu\nu}$  is the field strength, and  $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\lambda\sigma}F^{\lambda\sigma}$  its dual. Here  $N$ ,

$$N = \frac{\alpha}{8\pi} \int d^4x \operatorname{tr} \left( F_{\mu\nu} \tilde{F}^{\mu\nu} \right) \quad (2)$$

is the winding number of an Euclidean field configuration [1,2,3,4,5].

Instantons are solutions of the field equation with finite action and nonzero topological charge,  $N \neq 0$ . For the  $SU(3)$  color gauge field, the physics of such configurations are well known. One can add a term such as Eq. (1) to the action, which violates CP symmetry. A small value of  $\theta$  can be attained by using the Peccei-Quinn symmetry [6], and coupling to an axion field [7].

In the electroweak theory, for the  $SU(2)_L$  gauge field, the associated  $\theta$  angle has no physical significance. Since the electroweak action conserves baryon number, it does not change under a rotation of  $(B + L)$  number, where  $B$  and  $L$  denote baryon and lepton number, respectively. The only place where electroweak instantons contribute are in amplitudes connecting states with

different numbers of baryons [1,4]. For such processes, if  $(B + L)$  changes by  $\Delta(B + L) = 3N$ , then  $\theta$  appears in the path integral as  $e^{iN\theta}$ . The factor of three arises here because each generation of quark is produced. The basic instanton process therefore involves 9 colored quarks and three leptons. In amplitudes squared, the phase disappears and the  $\theta$ -angle is no consequence.

## 2 Beyond the Standard Model

In generalizations of the standard model, one can have processes that violate  $(B + L)$  explicitly. Following Anselm and Johansen [8], let us assume there is an explicit  $(B + L)$  violating interaction of the form

$$S_{(B+L)} = \frac{1}{M^2} \int d^4x \{ \lambda l_L q_L q_L q_L + c.c. \}. \quad (3)$$

Here  $l_L$  is a left handed lepton field and  $q_L$  is a left handed quark field. The scale  $M$  is the energy scale at which lepton and baryon number changing interactions are important, and is presumably a scale of a Grand Unified Theory (GUT) or higher. The matrix  $\lambda$  is of order 1, and contracts various spinor, color and flavor indices. This interaction violates both  $(B + L)$  and chirality.

The basic process that can generate vacuum to vacuum overlap is shown in Fig. (1). A  $SU(2)_L$  instanton emits three baryons and one lepton per generation; this is compensated by vertices for the  $(B + L)$  process of Eq. (3), so that in all, the total amplitude does not change  $(B + L)$ .

One might expect that since the scale of explicit  $(B + L)$  violation in Eq. (3) is much larger than the electroweak scale, that surely integrating over the instanton scale size cuts off such efforts. This is wrong. After integrating over the external lines of quarks and leptons, the instanton amplitude is

$$I = \int d^4x \int \frac{d\rho}{\rho^5} \frac{1}{\rho^6 M^6} e^{-2\pi/\alpha(\rho)}. \quad (4)$$

For the electroweak theory, the coupling constant decreases somewhat at high momentum, or small  $\rho$ ,

$$\frac{1}{\alpha(\rho)} = \frac{1}{\alpha(\rho_0)} + \frac{(22/3 - N_f/3 - N_h/6)}{2\pi} \ln(\rho_0/\rho), \quad (5)$$

where  $N_h$  is the number of Higgs particles and  $N_f$  is the number of electroweak doublets. Using the values for the standard model,  $N_f = 12$  and  $N_h = 1$ , we obtain  $22/3 - N_f/3 - N_h/6 = 19/6$ . This means that for small instantons,  $\rho \gg 1/M$ , the integration over instanton scale size behaves as

$$\sim \frac{1}{M^6} \int \frac{d\rho}{\rho^{47/6}}. \quad (6)$$

This integral does not converge, even for very small scale instantons above the scale  $M$ . Presumably above this scale, new physics enters which makes the  $\rho$  integral convergent.

This lack of sensitivity to the low energy integration also occurs in supersymmetric GUT's [9,10]. In such theories, the electroweak coupling actually increases until the unification scale, with the  $\rho$  integral convergent only above the GUT scale. Thus in supersymmetric GUT's, the integration over instanton scale size is less convergent than without supersymmetry.

We conclude that whatever physics is associated with a  $SU(2)_L$  theta angle and vacuum to vacuum transitions in the electroweak theory is sensitive to physics at super high energy scales.

Let us assume that the scale of explicit chiral  $(B + L)$  violation is the scale at which new physics makes the integral over  $\rho$  converge. This scale is  $M$ . It could be a scale somewhat less than that

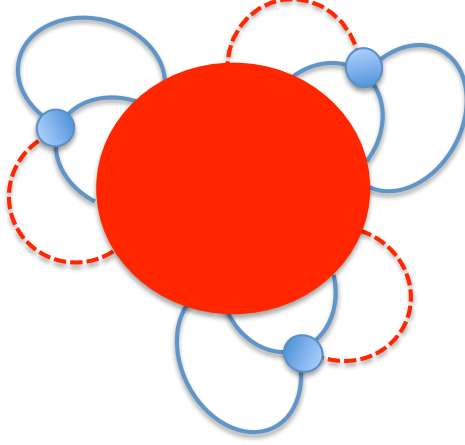


Figure 1: The instanton decays into 9 quarks (solid blue lines) and three leptons (dashed red line). They annihilate at the vertex associated with explicit  $(B + L)$  violation and chirality violation.

of the Planck scale, where Grand Unification might occur, or it might be the Planck scale itself. Including the effects of zero modes [8], the rate for instanton processes is

$$S_I = \kappa \left( \frac{2\pi}{\alpha_W} \right)^4 e^{-2\pi/\alpha(M)} M^4, \quad (7)$$

where  $\kappa$  is a constant of order 1. If we use electroweak theory with no Grand Unification, this formula becomes

$$S_I = \kappa \left( \frac{2\pi}{\alpha_W} \right)^4 \left( \frac{M_{EW}}{M} \right)^{19/6} e^{-2\pi/\alpha_W(M_{EW})} M^4. \quad (8)$$

If we take the energy scale  $M$  to be the Planck mass, and  $1/\alpha_W \sim 1/29$ , we find that

$$S_I \sim 10^{-122} \cdot M_{pl}^4. \quad (9)$$

This is remarkably close to the value of dark energy presently observed cosmologically,  $\epsilon_{DE} \sim 10^{-123} \cdot M_{pl}^4$  [11]. In Eq. (9),  $M_{pl} \sim 10^{19}$  GeV is the Planck mass, and not the reduced mass,  $M_{pl}/\sqrt{8\pi}$ .

Note that in Grand Unified theories, the coupling decreases less rapidly, or increases at higher energies, making the exponential suppression less important. At the Planck scale, one expects that this result is larger than the vacuum energy, although one might argue that in a GUT, one would generically go to a scale lower in energy.

An estimate similar to ours has been performed in a supersymmetric theory by Nomura, Watari, and Yanagida [9], see also Ref. [10]. In supersymmetric GUT theories, the electroweak coupling increases more rapidly than without supersymmetry. Consequently, to obtain an energy density as above, it is necessary to have a low value of the GUT scale [9,10]. The purpose of this note is to show that the numbers in a model *without* supersymmetry are extremely interesting in their own right.

Tegmark, Aguirre, Rees, and Wilczek, and later Hertzberg, Tegmark, and Wilczek [12], consider axion models with an axion scale on order of the Planck mass. They also find a reasonable value for the dark energy from the axion potential. Their model does not involve electroweak axions, though, and so differ in some important details from ours.

There are numerical factors that need to be computed, in order to get a more precise estimate of instanton induced processes within the electroweak theory. There is a coefficient associated with the precise normalization of zero modes in the one loop computation [8]. We have checked that using the running of  $\alpha_W$  to two loop order changes the exponent in Eq. (9) by  $\sim 1\%$ . Given all the uncertainty, the estimate we have obtained is remarkably close to the observed value of the dark energy.

It is interesting to consider how our estimate changes by altering the matter of the theory. Adding a single Higgs field has relatively little effect,  $\sim 10^{-3}$ . In contrast, a fourth generation contributes three quarks and one lepton, and so suppresses the estimate in Eq. (9) by a significant factor,  $\sim 10^{-21}$ .

One might worry that even though the instanton amplitude contributes to the energy of the  $\theta$ -vacuum ground state, that the rate for any such process, which involves square of the amplitude, is so small that it never happens in the lifetime of the universe. In the evolution of the universe from its initial conditions, though, it should suffice that there be an overlap between the initial set of states and a  $\theta$ -vacua. Since the lifetime of the universe, times the splitting of energy between the states include instantons and those which do not include instantons, is large, presumably one projects onto the proper ground state. If so, electroweak instantons to contribute to the ground state energy as above.

### 3 Electroweak Axions

If we promote the electroweak  $\theta$ -angle to an axion [6,7] we generate a vacuum energy of order  $S_I$ . Let us take the axion field modulus,  $F_A \sim M$ , as is natural if there is one high energy scale. If  $M$  is the Planck or a GUT scale, a very small axion mass is generated. Such a light axion does not affect the long range gravitational force. The electroweak axion is a pseudo-scalar, so that due to derivative couplings, at large distances the exchange of the electroweak axion is suppressed by two powers of  $r$ , relative to the  $1/r$  potential of gravity.

In the absence of Grand Unification, it is natural to assume that  $M \sim M_{\text{pl}}$  [13,14], and the electroweak coupling runs up to the Planck scale without substantial modification. In this case, the Compton wavelength for the electroweak axion is larger than the size of the universe. This is automatic since  $S_I \sim \epsilon_{\text{vac}}$ , and  $M_A \sim 1/R_{\text{universe}}$  by Einstein's equations. For such a large value of  $F_A$ , the electroweak axion is very weakly coupled, and the lower bound on  $F_A$ , from the cooling of large stars by axion emission [15], is not a problem. There may be additional constraints on  $F_A$  where axion emission is competitive with gravitational radiation.

Because black holes have no hair, global symmetries can be broken by quantum effects at the Planck scale [16]. If we take a unification scale at the Planck scale, our model is sensitive to this effect. Having such a light axion mass may allow us to evade this criticism; in any case, we ignore it.

The argument that the energy density trapped in electroweak axions is the dark energy has a naturalness problem. There are other vacuum energy effects that are much larger, and must be artificially set to zero. We apply a naturalness condition:

- After the axion field has had time to relax to zero, the cosmological constant should vanish. Therefore at late times, the universe has vanishing cosmological constant.

If this condition is true, then the electroweak axion energy is non zero only because the modulus of the axion field has not yet relaxed to zero. Recall that the vacuum energy is

$$\epsilon_A \sim M_A^2 a^2 \sim M_A^2 F_A^2 \theta^2 \sim S_1 \theta^2. \quad (10)$$

Because the inverse axion mass is of the order of the present size of the universe, there has not been time for the axion field to relax to zero. This happens only at some very late time, where the axion energy becomes dynamical, and is ultimately a type of matter energy. Assuming that far in the future there is no cosmological constant, in the present epoch the vacuum energy in the electroweak axion field has physical significance. That is, the cosmological constant is a temporary phenomenon that awaits its ultimate decay.

## 4 Summary

Of course the picture we paint of the electroweak axion as a source of the cosmological constant is very speculative. Electroweak theory could have other intermediate energy scales that are important. There could be compensations and tuning of various energy scales so that within such a picture, one would get the correct dark energy.

It may also be that there is no electroweak axion, or that even if there is, the source of dark energy is not in the axion field.

Perhaps most interesting in these considerations is that instantons can be sensitive to physics in the far ultraviolet, as is also seen in the supersymmetric case [9,10]. There is no decoupling of high energy and low energy degrees of freedom. This, and the sensitivity of instantons to scale breaking effects suggests their importance in other contexts. Perhaps in technicolor theories where the running of the coupling constant is assumed to be quite slow, effects of broken symmetries on higher energy scale might be important [17,18].

It is also quite amusing that there is another indication that there may be no intermediate energy scale physics between the electroweak scale and that of the Planck scale. This comes from requiring that electroweak theory is sensible at intermediate scales, and that there are fixed points of coupling constant evolution at the Planck scale. These considerations lead to a prediction of the Higgs boson mass at 126 GeV [13]. Such a value is suggested by recent results from the LHC [19,20]. Schaposhnikov has argued that including massive right handed singlet neutrinos gives a reasonable cosmology for the standard model, with appropriate values for dark matter and the baryon asymmetry [21].

Lastly, we note that Zhitnitsky has argued that there may be other processes, related to instantons in strongly interactions, that might be responsible for the cosmological constant [22].

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## References

- [1] G. 't Hooft, Phys. Rev. D **14**, 3432 (1976) [Erratum-ibid. D **18**, 2199 (1978)].
- [2] C. G. Callan, Jr., R. F. Dashen and D. J. Gross, Phys. Lett. B **63**, 334 (1976); R. Jackiw and C. Rebbi, Phys. Rev. Lett. **37**, 172 (1976).
- [3] S. R. Coleman, Subnucl. Ser. **15**, 805 (1979).
- [4] N. V. Krasnikov, V. A. Rubakov and V. F. Tokarev, Phys. Lett. B **79**, 423 (1978).
- [5] G. 't Hooft, Phys. Rept. **142**, 357 (1986).
- [6] R. D. Peccei and H. R. Quinn, Phys. Rev. Lett. **38**, 1440 (1977); Phys. Rev. D **16**, 1791 (1977).
- [7] S. Weinberg, Phys. Rev. Lett. **40**, 223 (1978); F. Wilczek, *ibid.* **40**, 279 (1978);
- [8] A. A. Anselm and A. A. Johansen, Nucl. Phys. B **412**, 553 (1994) [arXiv:hep-ph/9305271].
- [9] Y. Nomura, T. Watari and T. Yanagida, Phys. Lett. B **484**, 103 (2000) [hep-ph/0004182].
- [10] F. Takahashi and T. T. Yanagida, Phys. Lett. B **635**, 57 (2006) [hep-ph/0512296].
- [11] P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. **75**, 559 (2003) [astro-ph/0207347].
- [12] M. Tegmark, A. Aguirre, M. Rees and F. Wilczek, Phys. Rev. D **73**, 023505 (2006) [astro-ph/0511774]; M. P. Hertzberg, M. Tegmark and F. Wilczek, *ibid.* **78**, 083507 (2008) [arXiv:0807.1726 [astro-ph]].
- [13] M. Shaposhnikov and C. Wetterich, Phys. Lett. B **683**, 196 (2010) [arXiv:0912.0208 [hep-th]].
- [14] M. Shaposhnikov, “New physics without new energy scale”, unpublished.
- [15] G. G. Raffelt, Phys. Rept. **198**, 1 (1990).
- [16] M. Kamionkowski and J. March-Russell, Phys. Lett. B **282**, 137 (1992) [arXiv:hep-th/9202003].
- [17] S. Dimopoulos and L. Susskind, Nucl. Phys. B **155**, 237 (1979).
- [18] T. W. Appelquist, D. Karabali and L. C. R. Wijewardhana, Phys. Rev. Lett. **57**, 957 (1986).
- [19] G. Aad *et al.* [ATLAS Collaboration], arXiv:1203.1303 [hep-ex].
- [20] S. Chatrchyan *et al.* [CMS Collaboration], arXiv:1202.1488 [hep-ex].
- [21] M. Shaposhnikov, Prog. Theor. Phys. **122**, 185 (2009).
- [22] F. R. Urban and A. R. Zhitnitsky, Nucl. Phys. B **835**, 135 (2010) [arXiv:0909.2684 [astro-ph.CO]]; Phys. Rev. D **82**, 043524 (2010) [arXiv:0912.3248 [astro-ph.CO]]; Phys. Rev. D **82**, 123532 (2011) [arXiv:1011.2425 [astro-ph.CO]]; A. R. Zhitnitsky, arXiv:1112.3365 [hep-ph]; E. Thomas and A. R. Zhitnitsky, arXiv:1203.6073 [hep-ph].